How to solve the Bridge and Torch problem using simulation

Martin Rupp
SCIENTIFIC AND COMPUTER DEVELOPMENT SCD LTD

Contents

1 Introduction **1**

2 Statement of the puzzle **1**

3 Definition of the simulation **2**

4 Programmation of the simulation **3**

5 Running the simulation **4**

6 Interest of using a simulation **4**

7 Annex **6**
7.1 Source code 1 6
7.2 Solution 1 8
7.3 Solution 2 10

1 Introduction

In this article, we will show an example of a puzzle that can be solved using algorithmic and computer simulation.

Puzzles are Common in recreational logic or mathematics. Some of them may be extremely hard to solve - or even impossible. Here we will consider an 'elementary' puzzle. That is to say, a puzzle where the space of the possible configurations is finite and where almost all of the data are known. We will show how to use elementary computer simulation to solve it.

2 Statement of the puzzle

This riddle is a classical riddle and is described as follows:

Let us suppose that four people A,B,C and D are located on the left bank of a river. The only way to cross the river is via an old bridge that can support only two people at a time. Finally, this is the night and it;s so dark that nobody can cross the bridge without a torch.

Knowing that there is only one torch, that A will cross the bridge in $T(A) = 1$ minute, B in $T(B) = 2$ minutes, $T(C) = 5$ minutes and $T(D) = 10$ minutes, is it possible that A,B,C and D go altogether to the right bank if the torch lasts only $T_{limit} = 17$ minutes?

If we name A,B,C and D as , respectively: '*young boy*', '*girl*', '*senior*' and '*old man*' so to fix the ideas. The first thing that will usually pop into the minds is that the young boy who is the fastest will carry the torch and bring individually with him each of the three remaining persons.

Such a trip would last $T = 2 \times T(A) + T(B) + T(C) + T(D) = 19 > T_{limit}$
Which means that the strategy - if it exists - is not obvious.

3 Definition of the simulation

We define a k^{th} round S_k by picking at start time $t = t_{1,k}$ a random person $P_k^\alpha \in L(k,t)|_{t=t_{1,k}}$ among the people on the left bank which will carry the torch and another random person $P_k^\beta \in L(k,t), P_k^\alpha \neq P_k^\beta|_{t=t_{1,k}}$ that will go with P_k^α through the old bridge to the left bank of the river. At the time t, $L(k,t)$ is the set of people on the left bank, $R(k,t)$ is the set of people on the right bank, $B(k,t)$ is the set of people on the bridge. Of course we have :

$L(k,t) \cup R(k,t) \cup B(k,t) = \{A, B, C, D\}, \forall t$

Finally we define $T(X), X \in \{A, B, C, D\}$ as the time needed to cross the bridge (in minutes)

At the time $t = t_{2,k}$ we have:

$L(k,t_{2,k}) = L(k,t_{1,k}) - \{P_k^\alpha, P_k^\beta\}$
$B(k,t_{2,k}) = \{P_k^\alpha, P_k^\beta\}$
$R(k,t_{2,k}) = R(k,t_{1,k})$

At the time $t = t_{3,k}$ we have:

$L(k,t_{3,k}) = L(k,t_{2,k})$
$B(k,t_{3,k}) = \{\emptyset\}$
$R(k,t_{3,k}) = R(k,t_{2,k}) + \{P_k^\alpha, P_k^\beta\}$

$t_{3,k} = t_{1,k} + max(T(P_k^\alpha), T(P_k^\beta))$

At time $t = t_{4,k}$, If $L(k,t_{2,k}) \neq \{\emptyset\}$ then we choose a random person to carry the torch back to the left bank by picking a person in $R(k,t_{2,k})$. This is possible, since $|R(k,t_{2,k})| \geq 2$
We get a random person $P_k^\gamma \in R(k,t)$ that carries the torch back. The sets remained of course unchanged by that operation.

At time $t = t_{5,k}$ P_k^γ crosses the bridge back

$L(k,t_{5,k}) = L(k,t_{4,k}) - P_k^\gamma$

$B(k, t_{5,k}) = \{P_k^\gamma\}$
$R(k, t_{5,k}) = R(k, t_{4,k})$

At time $t = t_{6,k}$ P_k^γ is on the left side of the bank

$L(k, t_{6,k}) = L(k, t_{5,k})$
$B(k, t_{6,k}) = \{\oslash\}$
$R(k, t_{6,k}) = R(k, t_{5,k}) + P_k^\gamma$

$t_{6,k} = t_{4,k} + T(P_k^\gamma)$

This is the completion of the Step S_k

Of course, we can assume that the operations of swapping the torches and picking people are done in a neglectable amount of time (one or two seconds) and so the time required for Step S_k is $T_k = max(T(P_k^\alpha), T(P_k^\beta) + T(P_k^\gamma)$ if S_k is not the last step and $T_k = max(T(P_k^\alpha), T(P_k^\beta)$ otherwise.

It is not difficult to see that there can be only 3 steps S_1, S_2, S_3so we have a global completion time of $T = T_1 + T_2 + T_3$
That is to say:

$$T = max(T(P_1^\alpha), T(P_1^\beta)) + T(P_1^\gamma) + max(T(P_2^\alpha), T(P_2^\beta)) + T(P_2^\gamma) + max(T(P_3^\alpha), T(P_3^\beta))$$

The program is therefore described as an optimization program as such:

$$MIN[max(T(P_1^\alpha), T(P_1^\beta)) + T(P_1^\gamma) + max(T(P_2^\alpha), T(P_2^\beta)) + T(P_2^\gamma) + max(T(P_3^\alpha), T(P_3^\beta))]$$
With the following constraints:

- $(P_1^\alpha, P_1^\beta) \in \{A, B, C, D\}, P_1^\alpha \neq P_1^\beta$
- $P_1^\gamma \in \{P_1^\alpha, P_1^\beta\}$
- $(P_2^\alpha, P_2^\beta) \in \{A, B, C, D\} - (\{P_1^\alpha, P_1^\beta\} - P_1^\gamma), P_2^\alpha \neq P_2^\beta$
- $P_2^\gamma \in \{P_2^\alpha, P_2^\beta\} + (\{P_1^\alpha, P_1^\beta\} - P_1^\gamma)$
- $(P_3^\alpha, P_3^\beta) \in P_2^\gamma + (\{A, B, C, D\} - (\{P_1^\alpha, P_1^\beta\} - P_1^\gamma) - P_2^\gamma)$
- $T(A) = 1$
- $T(B) = 2$
- $T(C) = 5$
- $T(D) = 10$

4 Programmation of the simulation

We chose to use the PERL language for the simulation.
First, we define a hash table where the characteristics of A, B, C, D are stored.

my %g_people=('young boy'=>'L','girl'=>'L','senior'=>'L','old man'=>'L');
my %g_times =('young boy'=>1,'girl'=>2,'senior'=>5,'old man'=>10);

The hash *%g_people*will store the location of the people (L for left bank and R for right bank) while the hash *%g_times* will store the values of $T(A), T(B), T(C), T(D)$.

We define two simple functions *peopleOnLeftSide()* and *numberOfPeopleOnLeftSide()* which will respectively return the list of people on the left bank and their amount.

We will use a random generator *rand(n)* that will return a number between 0 and n-1.

We will code two loops. The first loop will iterate the tries, until a solution can be founded, and the second loop will iterate the 3 steps S_1, S_2, S_3.

In order to simulate the displacement of some people, namely *$key* , from a bank to another, we simply change the *%g_people{$key}* value.

We also code a function *printSituation()* that will print on a console a visualisation of the situation at any moment, displaying the people who are on the left and right banks.

The complete listing of the code used for the simulation can be found in the annex (**Source code 1**)

5 Running the simulation

We run the simulation and a solution is quickly exhibed, which consists in swapping adequately people on the right side of the bank at the second step S_2. If rigorously implemented, the simulation isn't long to run and can even provide all the possible solutions.

We find that the puzzle has (at least) one solution (See **Solution 1** in annex).

If we try for 16 minutes, we do not find a solution, meaning 17 minutes is the best time that could be achieved[1].

6 Interest of using a simulation

The interest of using a simulation approach (versus trying to solve the puzzle by using mental computations or mathematics) is that there is a sure path to the solution. If the simulation is

[1]To be completely rigorous, since we use simulation with random variables, we can only mention that it is extremely unlikely that a solution with 16 minutes could exist, however we could easily change the code to make sure we have covered all the cases.

correctly coded, then, the program will find a solution *if* such a solution does exist.

It is also possible to find more solutions, even all of them and to extend the original puzzle. For example, it is quite easy to compute solution of a more general problem, when there are n people $X_1..., X_n$on the left side of the bridge with different times $T(X_1)..., T(X_n)$ and to compute the best possible time.

It is easy to compute the maximal amount of combination we need to perform to have all the possible cases, it is $N(n) = \prod_{x+y=n+2, x>2} C_2^x \times C_1^y$.

In the case of n=4 people, the maximal amount of cases to test is $N(4) = C_2^4 \times C_1^2 \times C_2^3 \times C_1^3 = 10 \times 2 \times 3 \times 3 = 180$.

For instance, the simulation can now provide solutions to infinite variants of the puzzle:

1. If there is another young boy who is also able to cross the bridge in T=1 minute, can the time of 17 minutes still be achieved?

   Answer is 'yes'(See **Solution 2** in annex).

1. If there are 1 young boy, 2 girls, 2 seniors and 1 old man, can the time of 26 minutes still be achieved?

Answer is 'yes'. We can represent the solution as a sequence of displacements through the bridge.

girl1 young_ boy $\rightarrow$
$\leftarrow$ *girl1*
old_ man senior2 $\rightarrow$
$\leftarrow$ *young_ boy*
girl2 young_ boy $\rightarrow$
$\leftarrow$ *young_ boy*
girl1 young_ boy $\rightarrow$
$\leftarrow$ *young_ boy*
young_ boy senior1 $\rightarrow$

Note that in the case of 10 people, we have:$N(10) = C_2^{10} \times C_1^2 \times C_2^9 \times C_1^3 \times C_2^8 \times C_1^4 \times C_2^7 \times C_1^5 \times C_2^6 \times C_1^6 \times C_2^5 \times C_1^7 \times C_2^4 \times C_1^8 \times C_2^3 \times C_1^9$
$= 933295426560000$.

This means that a simulation can take some time before a solution is found if the amount of people is too big.

7 Annex

7.1 Source code 1

```
use strict;
use List::Util qw(min max);
#globals
my %g_people=('young_boy'=>'L','girl'=>'L','senior'=>'L','old_man'=>'L');
my %g_times =('young_boy'=>1,'girl'=>2,'senior'=>5,'old_man'=>10);
my $g_limit=17;
my $g_T=0;

sub peopleOnLeftSide()
{
    my @ppl;
    foreach my $keys (keys %g_people)
    {

        if ($g_people{$keys} eq 'L')
        {
            push @ppl, $keys;
        }
    }
    return @ppl;
}


sub printSituation()
{
    print "\n\n";
    foreach my $keys (keys %g_people)
    {

        if ($g_people{$keys} eq 'L')
        {
            print $keys," ";
        }
    }
    print "\t\t\t\t";
    foreach my $keys (keys %g_people)
    {
        if ($g_people{$keys} eq 'R')
        {
            print $keys," ";
        }
    }
    print "\n———————————————————————————-";
    print "\n———————————————————————————-";
    print "\n||          ||            ||           ||            ||\n";
    print "~~~~~~   ~~ ~~ ~ ~    ~~~~ ~~ ~~~   ~~~~~~~~~~~~~   ~~~~  ~~ ~~~~\n";
    print "~~~~~~       ~~~~~ ~~~~ ~~~   ~~~~~~   ~~~~ ~ ~ ~ ~ ~ ~~~~~ ~~~\n";
```

```
    print "\n\n";
}


sub numberOfPeopleOnLeftSide()
{
    my @peoples= peopleOnLeftSide();
    return ($#peoples+1);
}

print "initial number on left side=",numberOfPeopleOnLeftSide(),"\n";

$g_T=20;

while($g_T>$g_limit)
{
    $g_T=0;
    %g_people=('young_boy'=>'L','girl'=>'L','senior'=>'L','old_man'=>'L');
while(1)
{

    my $N_L=numberOfPeopleOnLeftSide();
    my @peoples=peopleOnLeftSide();

    #pick up a random bearer of the torch
    my $r1= int rand($N_L);

    my $r2= int rand($N_L);

    #pickup a random walker
    while($r2==$r1)
    {
        $r2= int rand($N_L);
    }

    my $T1=$g_times{$peoples[$r1]};
    my $T2=$g_times{$peoples[$r2]};

    my $t_trip=max($T1,$T2);

    $g_T+= $t_trip;

    print "$peoples[$r1] is going to the right side with $peoples[$r2] \n";
    print "Trip to right side took $t_trip minutes \n";

    $g_people{$peoples[$r2]}='R';
    $g_people{$peoples[$r1]}='R';

    printSituation();

    $N_L=numberOfPeopleOnLeftSide();
```

```
    if($N_L==0)
    {
        printSituation();
print "everybody is on the Right side \n";
        last;
    }
    else
    {
    print "$N_L are still on the Left side \n";
    my $N_R=4-$N_L;
      #we may switch torch with the other person already there
        my $r3= int rand($N_R);
            #we switch
            my $i=0;
            foreach my $key (keys %g_people)
                {

                    if ($g_people{$key} eq 'R')
                    {

                        if($i==$r3)
                        {
                        $g_people{$key}='L';
                print "$key is going back to the left side with the torch \n";
                        my $T3=$g_times{$key};
                        print "Return trip took $T3 minutes \n";
                        #return time
                        $g_T+= $T3;
                        }
                        $i++;
                    }
                }
    printSituation();
        }
}
print "————————————————————————-\n";
print "**** it took $g_T minutes in total ****\n";
print "————————————————————————-\n";
}
```

7.2 Solution 1

picked up randomly 0 and 2
young_boy is going to the right side with girl
Trip to right side took 2 minutes

old_man senior *young_boy girl*

——————————————————————————————-

——————————————————————————————-

|| || || || ||

~~~~~~ ~~ ~~ ~ ~ ~~~~ ~~ ~~~ ~~~~~~~~~

~~~~~~ ~~~~~ ~~~~ ~~~ ~~~~~~ ~~~~ ~

2 are still on the Left side
young_boy is going back to the left side with the torch
Return trip took 1 minutes

young_boy old_man senior *girl*

——————————————————————————————-

——————————————————————————————-

|| || || || ||

~~~~~~ ~~ ~~ ~ ~ ~~~~ ~~ ~~~ ~~~~~~~~~

~~~~~~ ~~~~~ ~~~~ ~~~ ~~~~~~ ~~~~ ~

picked up randomly 1 and 2
old_man is going to the right side with senior
Trip to right side took 10 minutes

young_boy *old_man girl senior*

——————————————————————————————-

——————————————————————————————-

|| || || || ||

~~~~~~ ~~ ~~ ~ ~ ~~~~ ~~ ~~~ ~~~~~~~~~

~~~~~~ ~~~~~ ~~~~ ~~~ ~~~~~~ ~~~~ ~

1 are still on the Left side
girl is going back to the left side with the torch
Return trip took 2 minutes

young_boy girl *old_man senior*

——————————————————————————-

——————————————————————————-

|| || || || ||

∼∼∼∼∼∼ ∼∼ ∼∼ ∼ ∼ ∼∼∼∼ ∼∼ ∼∼∼ ∼∼∼∼∼∼∼∼∼

∼∼∼∼∼∼ ∼∼∼∼∼ ∼∼∼∼ ∼∼∼ ∼∼∼∼∼∼ ∼∼∼∼ ∼

picked up randomly 0 and 1
young_boy is going to the right side with girl
Trip to right side took 2 minutes

young_boy old_man girl senior

——————————————————————————-

——————————————————————————-

|| || || || ||

∼∼∼∼∼∼ ∼∼ ∼∼ ∼ ∼ ∼∼∼∼ ∼∼ ∼∼∼ ∼∼∼∼∼∼∼∼∼

∼∼∼∼∼∼ ∼∼∼∼∼ ∼∼∼∼ ∼∼∼ ∼∼∼∼∼∼ ∼∼∼∼ ∼

everybody is on the Right side
——————————————————————————-
∗∗∗∗ *it took 17 minutes in total* ∗∗∗∗
——————————————————————————-

7.3 Solution 2

young_boy is going to the right side with other_young_boy
Trip to right side took 1 minutes

girl old_man senior *other_young_boy young_boy*

——————————————————————————-

——————————————————————————-

|| || || || ||

∼∼∼∼∼∼ ∼∼ ∼∼ ∼ ∼ ∼∼∼∼ ∼∼ ∼∼∼ ∼∼∼∼∼∼∼∼∼∼∼∼ ∼∼∼∼ ∼∼ ∼∼∼∼

∼∼∼∼∼∼ ∼∼∼∼∼ ∼∼∼∼ ∼∼∼ ∼∼∼∼∼∼ ∼∼∼∼ ∼ ∼ ∼ ∼ ∼ ∼∼∼∼∼ ∼∼∼

3 are still on the Left side
other_young_boy is going back to the left side with the torch
Return trip took 1 minutes

other_young_boy girl old_man senior *young_boy*

——————————————————————————-

——————————————————————————-

|| || || || ||

∼∼∼∼∼∼ ∼∼ ∼∼ ∼ ∼ ∼∼∼∼ ∼∼ ∼∼∼ ∼∼∼∼∼∼∼∼∼∼∼∼ ∼∼∼∼ ∼∼ ∼∼∼∼

∼∼∼∼∼∼ ∼∼∼∼∼ ∼∼∼∼ ∼∼∼ ∼∼∼∼∼∼ ∼∼∼∼ ∼ ∼ ∼ ∼ ∼ ∼∼∼∼∼ ∼∼∼

senior is going to the right side with old_man
Trip to right side took 10 minutes

other_young_boy girl *young_boy old_man senior*

——————————————————————————-

——————————————————————————-

|| || || || ||

∼∼∼∼∼∼ ∼∼ ∼∼ ∼ ∼ ∼∼∼∼ ∼∼ ∼∼∼ ∼∼∼∼∼∼∼∼∼∼∼∼ ∼∼∼∼ ∼∼ ∼∼∼∼

∼∼∼∼∼∼ ∼∼∼∼∼ ∼∼∼∼ ∼∼∼ ∼∼∼∼∼∼ ∼∼∼∼ ∼ ∼ ∼ ∼ ∼ ∼∼∼∼∼ ∼∼∼

2 are still on the Left side
young_ boy is going back to the left side with the torch
Return trip took 1 minutes

other_ young_ boy girl young_ boy *old_ man senior*

—————————————————————————————-

—————————————————————————————-

|| || || || ||

∼∼∼∼∼∼ ∼∼ ∼∼ ∼ ∼ ∼∼∼∼ ∼∼ ∼∼∼ ∼∼∼∼∼∼∼∼∼

∼∼∼∼∼∼ ∼∼∼∼∼ ∼∼∼∼ ∼∼∼ ∼∼∼∼∼∼ ∼∼∼∼ ∼

young_ boy is going to the right side with other_ young_ boy
Trip to right side took 1 minutes

girl *other_ young_ boy young_ boy old_ man senior*

—————————————————————————————-

—————————————————————————————-

|| || || || ||

∼∼∼∼∼∼ ∼∼ ∼∼ ∼ ∼ ∼∼∼∼ ∼∼ ∼∼∼ ∼∼∼∼∼∼∼∼∼

∼∼∼∼∼∼ ∼∼∼∼∼ ∼∼∼∼ ∼∼∼ ∼∼∼∼∼∼ ∼∼∼∼ ∼

1 are still on the Left side
young_ boy is going back to the left side with the torch
Return trip took 1 minutes

girl young_ boy *other_ young_ boy old_ man senior*

—————————————————————————————-

—————————————————————————————-

|| || || || ||

∼∼∼∼∼∼ ∼∼ ∼∼ ∼ ∼ ∼∼∼∼ ∼∼ ∼∼∼ ∼∼∼∼∼∼∼∼∼

~~~~~~ ~~~~~ ~~~~ ~~~ ~~~~~~ ~~~~ ~

girl is going to the right side with young_ boy
Trip to right side took 2 minutes

other_ young_ boy girl young_ boy old_ man senior

——————————————————————————-

——————————————————————————-

|| || || || ||

~~~~~~ ~~ ~~ ~ ~ ~~~~ ~~ ~~~ ~~~~~~~~~

~~~~~~ ~~~~~ ~~~~ ~~~ ~~~~~~ ~~~~ ~

everybody is on the Right side
——————————————————————————-
∗∗∗∗ *it took 17 minutes in total* ∗∗∗∗
~~~~~~